Geometry (G.C.5)
Unit Five: Arc Length (IC16)

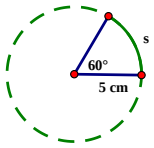
Name: _____
Date: _____ Period: _____

Arc Length:

The distance associated with a portion of a circle's circumference; can be abbreviated as "S" or as $\overset{\frown}{AB}$

* Units are the same as the radius units*

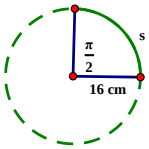
In Degrees:



$S = \frac{x}{360} \cdot 2\pi r$

Where x is the central angle/arc's measure

In Radians:



$S = \Theta r$

Where Θ is the central angle measure and r is the circle's radius.

1. Determine the arc length.

a) Central Angle of 30°, radius of 3 cm

$S = \frac{30}{360} \cdot 2\pi\,(3) = \frac{180\pi}{360}$

s = $\frac{\pi}{2}$ cm **(E)**

Why???

$S = \frac{\Theta}{2\pi} \cdot 2\pi r = \Theta r$

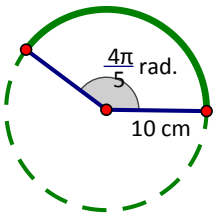
b) Central Angle of $\frac{\pi}{4}\ rad.$**, radius of 12 cm**

$S = \frac{\pi}{4}(12) = \frac{12\pi}{4}$

s = 3π cm **(E)**

2. Determine the arc length of the following.

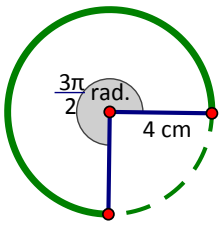
a)



$S = \frac{4\pi}{5}(10) = \frac{40\pi}{5}$

s = 8π cm **(E)**

b)



$S = \frac{3\pi}{2}(4) = \frac{12\pi}{2}$

s = 6π cm **(E)**

3. Determine the missing information.

a) s = 10π cm, r = 8 cm

$S = \Theta r$

$10\pi = \Theta(8)$

$\frac{10\pi}{8} = \Theta$

Θ = $\frac{5\pi}{4}$ rad.

b) Θ = $\frac{2\pi}{5}$ rad., s = 5π cm

$S = \Theta r$

$5\pi = \frac{2\pi}{5} \cdot r$

$25\pi = 2\pi r$

r = 12.5 cm

c) r = 8 cm, Θ = $\frac{\pi}{2}$ rad.

$S = \Theta r$

$S = \frac{\pi}{2}(8) = \frac{8\pi}{2}$

s = 4π cm

d) Θ = $\frac{5\pi}{6}$ rad., s = 10π cm

$S = \Theta r$

$10\pi = \frac{5\pi}{6} \cdot r$

$6 \cdot 10\pi = \frac{5\pi}{6} \cdot r \cdot 6$

$60\pi = 5\pi r$

r = 12 cm

What's the difference between **arc length** and **arc measure**?

Arc length = distance around portion of a circle
Arc measure = degree measure of arc as a portion of 360°

4. Determine the information below if the radius of the circle is 5cm.

a) $m\widehat{CB}$= 50°

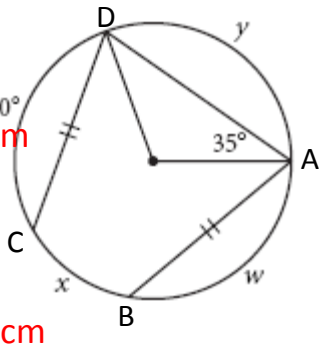
b) $m\widehat{BA}$= 100°

c) $m\widehat{AD}$= 110°

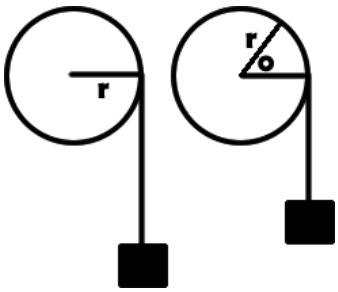
d) $\ell\widehat{CB}=\frac{50}{360}2\pi(5)=\frac{500\pi}{360}=\frac{25\pi}{18}$ cm

e) $\ell\widehat{BA}=\frac{100}{360}2\pi(5)=\frac{1000\pi}{360}=\frac{25\pi}{9}$ cm

f) $\ell\widehat{AD}=\frac{110}{360}2\pi(5)=\frac{1100\pi}{360}=\frac{55\pi}{18}$ cm



5. If the radius of the pulley is 12 cm and the rotation of the pulley was $\frac{7\pi}{6}$ radians, how many cm will the weight rise?



Arc length = how much is wrapped around = how much weight rises

$S=\frac{7\pi}{6}(12)=\frac{84\pi}{6}=14\,\pi$ cm

6. The rotation of the smaller gear with radius 10 cm was $\frac{11\pi}{6}$ radians. What was the angle of rotation (radians) of the larger gear with a radius 20 cm?

Arc length should match

$Small$: $\frac{11\pi}{6}(10)=\frac{110\pi}{6}=\frac{55\pi}{3}\pi$ cm

Big: $\frac{55\pi}{3}=\Theta(20)$

$\Theta=\frac{11\pi}{12}rad$