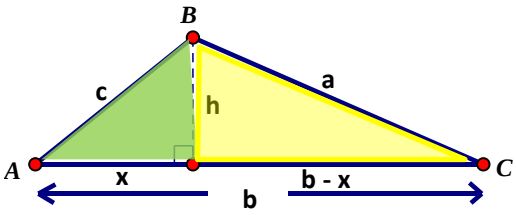
Geometry (G.SRT.10)
Unit Two: Law of Cosines (IC31)

Name: _____
Date: _____ Period: _____

1. Derive the Law of Cosines.



Green triangle

$x^2 + h^2 = c^2 \rightarrow x^2 = c^2 - h^2$

$\cos A = \frac{x}{c} \rightarrow x = c \cos A$

Yellow triangle

$h^2 + (b - x)^2 = a^2$ ← simplify this eqn

$h^2 + (b - x)(b - x) = a^2$

$h^2 + b^2 - 2bx + x^2 = a^2$

$h^2 + b^2 - 2bx + c^2 - h^2 = a^2$

$b^2 - 2bx + c^2 = a^2$

$b^2 - 2bc \cos A + c^2 = a^2$

$\boxed{a^2 = b^2 + c^2 - 2bc \cos A}$

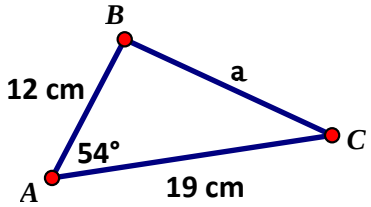
2. Why doesn't SSS and SAS information work with the Law of Sines?

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

No side/angle pair, so too many variables left to solve easily

3a) Using the Law of Cosines, solve for side a.



$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 19^2 + 12^2 - 2(19)(12)\cos 54$

$\sqrt{a^2} = \sqrt{19^2 + 12^2 - 456 \cos 54}$

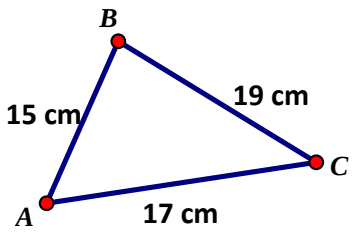
$a \approx 15.39$ cm

b) The teacher asks Jeremy to solve for the smallest angle next. Which angle is the smallest? ∠C

c) How can you determine which angle is smaller of the two?

Small angles are opposite of small sides since 12 cm < 19 cm, ∠C is less than ∠B.

4. Jazmine is about to solve this triangle by using the Law of Cosines. The teacher asks Jazmine to solve for the largest angle first.



a) Which angle is the largest? ∠A

b) How can you determine which angle is the largest?

Opposite longest side.

c) Solve the triangle.

$a^2 = b^2 + c^2 - 2bc \cos A$

$19^2 = 17^2 + 15^2 - 2(17)(15)\cos A$

$361 = 289 + 225 - 510 \cos A$

$\frac{-153}{-510} = \frac{-510 \cos A}{-510}$

$\cos A = 0.3$

$A = \cos^{-1}(0.3)$

$A \approx 73°$

*Once you know an angle, Switch to Law of Sines to Finish the problem.

$\frac{\sin 73}{19} = \frac{\sin B}{17}$

$19 \sin B = 17 \sin 73$

$\sin B = \frac{17 \sin 73}{19}$

$B = \sin^{-1}\left(\frac{17 \sin 73}{19}\right)$

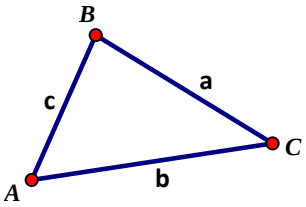
$B \approx 59°$

$m\angle C = 180 - 73 - 59 \approx 48°$

5. Why did we need the Law of Cosines? Why isn't the Law of Sines good enough?

We didn't have an angle/opp side pair.

6. Write out the three versions of the Law of Cosines for the given triangle.



$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$