Geometry (G.GMD.3) Name: ______________________________

Unit Three: Three Dimensional Solids (IC7-10) Date: _____________ Period: ___________

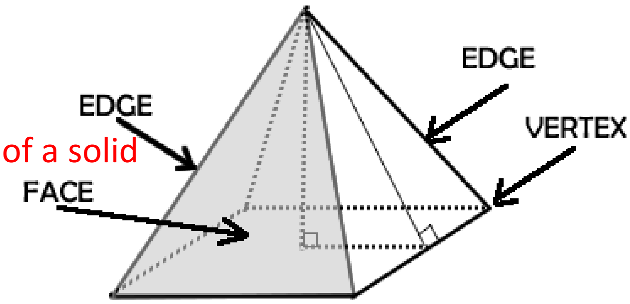
A Solid – 3-D closed figure

A Polyhedron – Solid with polygons as faces

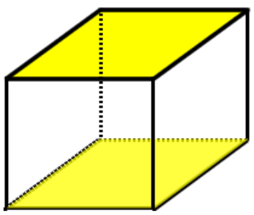
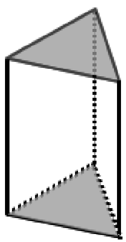
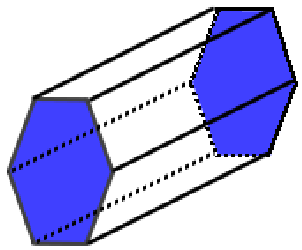
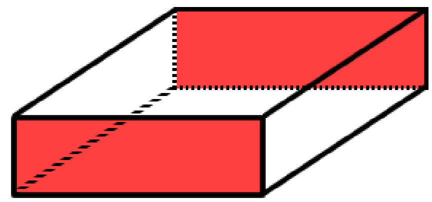
A Face of a Polyhedron – A polygon which is the "side" of a solid

An Edge – Intersection of 2 faces of solid

A Vertex – Intersection of edges of solid



PRISMS A solid formed by a polygon and its parallel, translated image being connected by quadrilaterals along their edges.

Bases of a prism – The ≅ and parallel faces of a prism (non-rectangular if present)

Lateral faces of a prism – Faces that are not the bases/ faces that connect the bases

Height of a prism – ⊥ distance between the 2 bases

Your Turn: Given the rectangular prism **with face BCFE as one of its bases**. Use each value ONLY ONCE.

__B or C__ 1. Edge

__E or A__ 2. Lateral Face

__A or E__ 3. Base

__D__ 4. Vertex

__B or C__ 5. Height

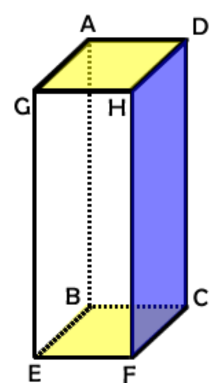
A. Rectangle ADHG

B. $\overline{HF}$

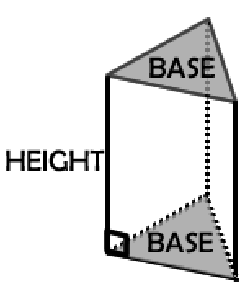
C. $\overline{AD}$

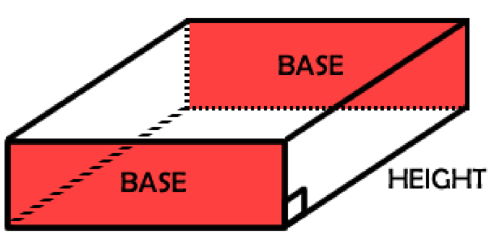
D. Point B

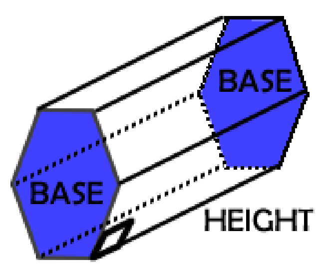
E. Rectangle HDCF



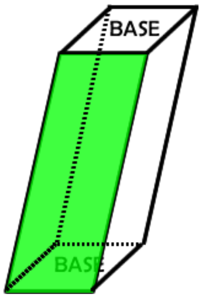
Right prisms – prisms with ⊥ bases and lateral faces

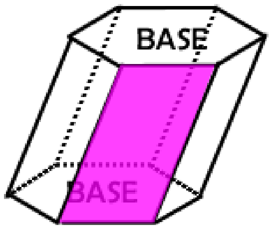


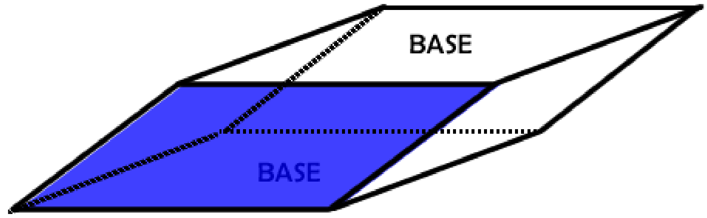




Oblique prisms – prisms with bases and lateral faces NOT ⊥



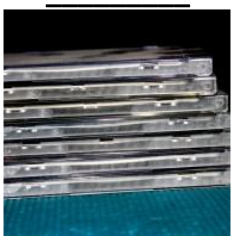


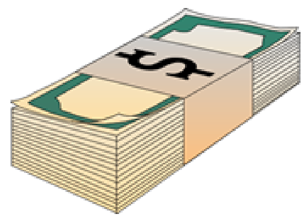


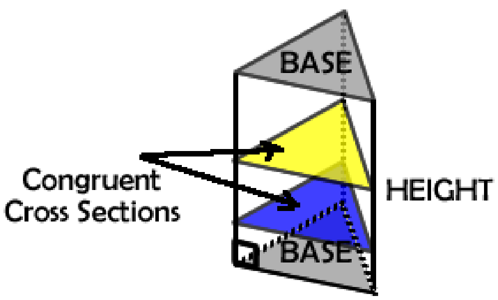
PRISM VOLUME – THE STACKING PRINCIPLE

Cross Section –

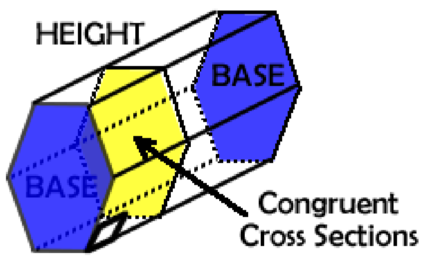
A Stack of CD Cases	A Stack of Paper	A Stack of Money	A Stack of Coasters
Cross Section: square	**Cross Section:** rectangle	**Cross Section:** rectangle	**Cross Section:** square



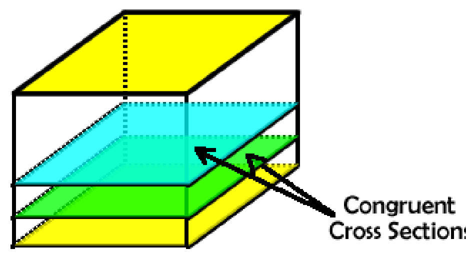




triangles



hexagons



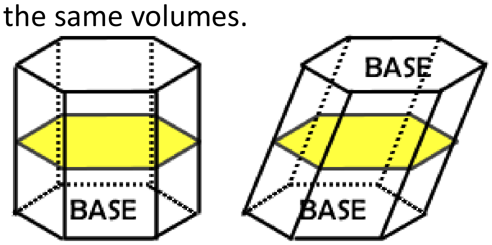
rectangles

The stacking of congruent parallel cross sections allows us to create a formula for the volume of prism.

Volume$_{PRISM}$ = Bh where B = the area of base and h = height of prism

Cavalieri's Principle: **If the areas of the cross sections of two solids by any plane parallel to a given plane are invariably equal, then the two solids have the same volume.**

In other words, if two prisms have the same height and the same base then oblique and right prisms will have the same volumes.

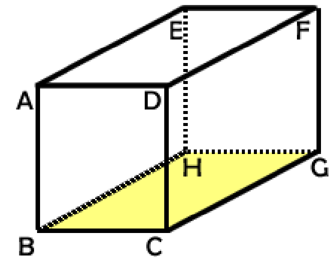


Volumes are equal.

Volumes are equal.

Your Turn:

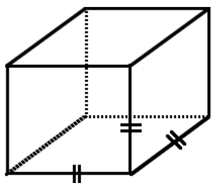
2. After looking at the rectangular prism to the right, a young lady in the class raises her hand and says, “Could I use rectangle ADCB as my base instead of rectangle BHGC?” What would you say?



Yes, because there is still one pair of ≅ and parallel faces

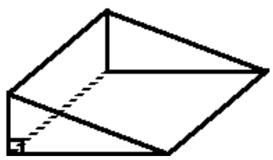
3. Properly name the following prisms.

a)



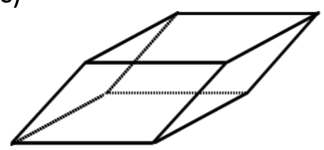
Name: cube

b)



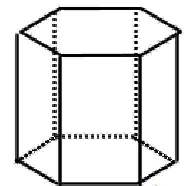
Name: Right triangular prism

c)



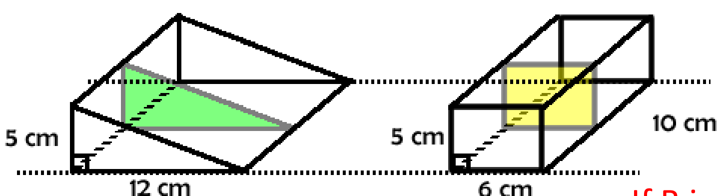
Name: Oblique rectangular prism

d)



Name: Right hexagonal prism

4. Jenny says that the two prisms DO NOT have the same volume because the cross sections are not the same. Renee disagrees; she says that it isn’t the shape that has to be the same it is the area. Renee thinks they have the same volume. Who is right and why? Find the volume of each prism to help justify your answer.

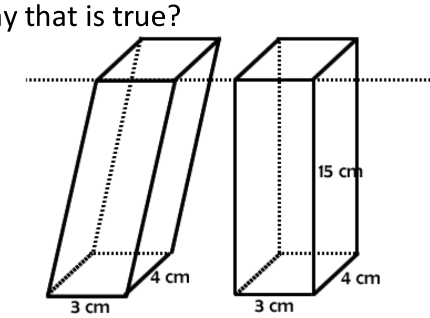


V = Bh
B = ½ (12)(5) = 30
V = (30)(10)
$V = 300\ cm^3$

V = Bh
B = (6)(5) = 30
V = (30)(10)
$V = 300\ cm^3$

If B is the same and h is the same, then = volume

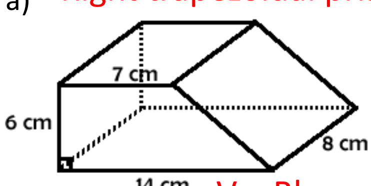
5. Cavalieri’s principle says that these two prisms have equal volume. Explain why that is true?



B = 12 for both and h = 15 for both, so = volume

6. Determine the volume of the prisms. (Lines that appear perpendicular are perpendicular.)

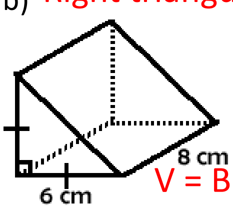
a) Right trapezoidal prism



V = Bh
B = ½ (6)(7 + 14) = 63
V = (63)(8)

Volume = $504\ cm^3$

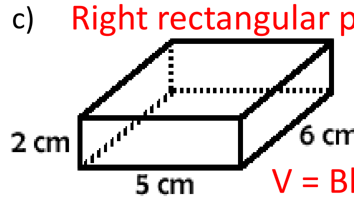
b) Right triangular prism



V = Bh
B = ½ (6)(6) = 18
V = (18)(8)

Volume = $144\ cm^3$

c) Right rectangular prism

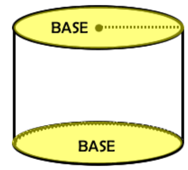


V = Bh
B = (2)(5) = 10
V = (10)(6)

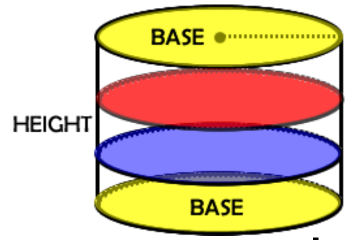
Volume = $60\ cm^3$

IC8 - CYLINDER VOLUME

Cross Sections –

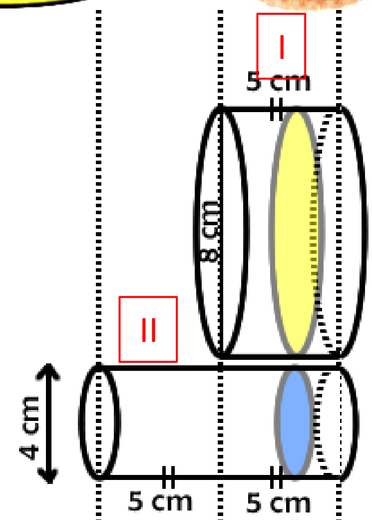


VOLUME$_{CYLINDER}$ = Bh
$= \pi r^2 h$



Your Turn:

1. Jared wants to test out a new theory….. Instead of having the cross area sections the same as Cavalieri suggested he wants to half the radius of one cross section and then double the height to make up for it. He believes because he divided the radius by 2 but doubled the height that the volumes should be equal. Is he correct? Explain.



I

$V = \pi r^2 h$

$= \pi(4)^2(5)$

$= 80\pi\ cm^3$

II

$V = \pi r^2 h$

$= \pi(2)^2(10)$

$= 40\pi\ cm^3$

* Not = b/c radius gets squared, but height does not.

2. The two solids below have the same volume and height. Find the measurement for the base edge of the prism.

$\pi r^2 h = Bh$

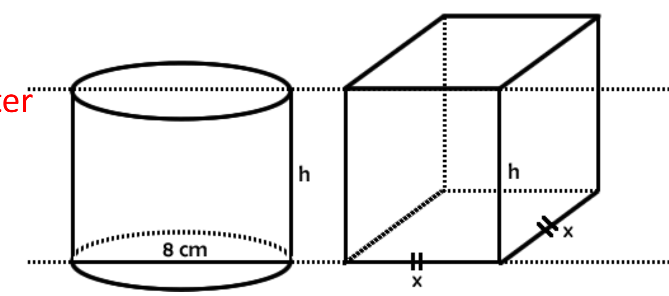
$\pi(4)^2 h = x^2 h$ → h is the same so it doesn't matter

$\pi(4)^2 = x^2$

$16\pi = x^2$

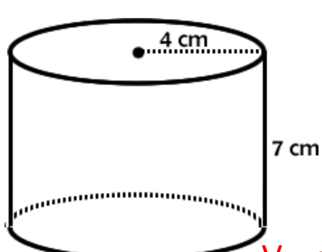
$\sqrt{16\pi} = \sqrt{x^2}$

$x \approx 7.1$ cm



3. Determine the volume of the cylinder.

a)

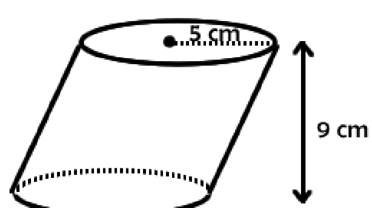


$V = \pi r^2 h$

$= \pi(4)^2(7)$

Volume = $112\pi\ cm^3$ (E)

b)



$V = \pi r^2 h$

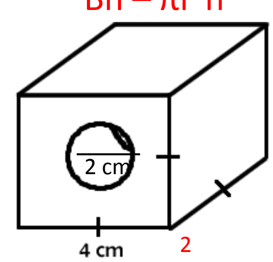
$= \pi(5)^2(9)$

Volume = $225\pi\ cm^3$ (E)

c)

Cube – cylinder

$Bh - \pi r^2 h$

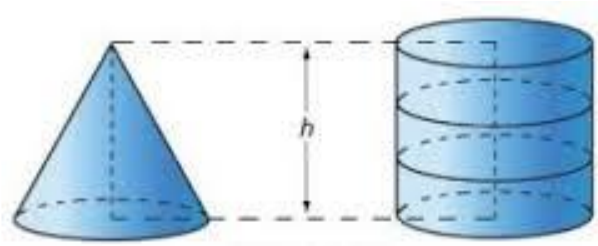


$16(4) - \pi(1)^2(4)$

Volume = $64 - 4\pi\ cm^3$ (E)

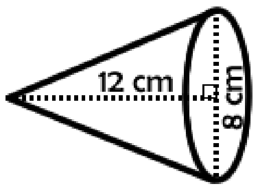
CONE VOLUME

VOLUME$_{CONE}$ = $\frac{Bh}{3}$ or $\frac{\pi r^2 h}{3}$



1) Determine the volume of each solid.

a)

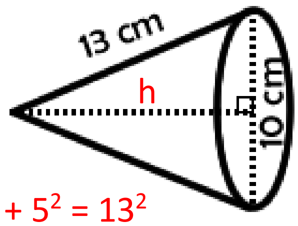


$V = \frac{\pi r^2 h}{3}$

$V = \frac{\pi(4)^2(12)}{3}$

$= 64\pi\ cm^3$

b)



$h^2 + 5^2 = 13^2$

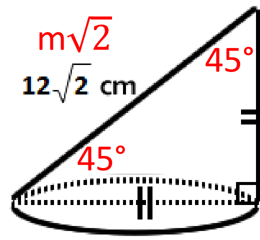
$h = 12$

$V = \frac{\pi r^2 h}{3}$

$V = \frac{\pi(5)^2(12)}{3}$

$= 100\pi\ cm^3$

c)

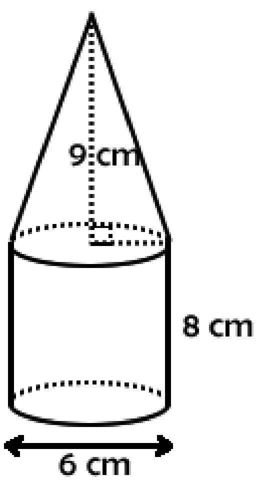


$V = \frac{\pi r^2 h}{3}$

$V = \frac{\pi(6)^2(12)}{3}$

$= 144\pi\ cm^3$

d)



2m

Cone + cylinder

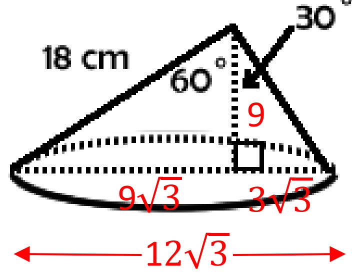
$\frac{\pi r^2 h}{3} + \pi r^2 h$

$\frac{\pi(3)^2(9)}{3} + \pi(3)^2(8)$

$27\pi + 72\pi$

$99\pi\ cm^3$

e)



$\sqrt{3}(9) = m\sqrt{3}(\sqrt{3})$

$\frac{9\sqrt{3}}{3} = \frac{3m}{3}$

$m = 3\sqrt{3}$

$V = \frac{\pi r^2 h}{3}$

$\frac{\pi(6\sqrt{3})^2(9)}{3}$

$324\pi\ cm^3$

IC 9 - PYRAMID VOLUME

Given the square pyramid.

___D___ 1. Slant Height

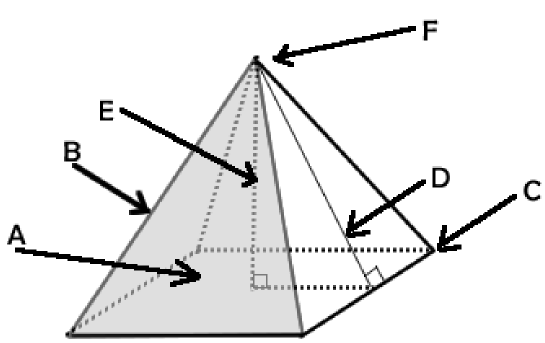
___F___ 2. Apex

___E___ 3. Height

___B___ 4. Lateral Edge

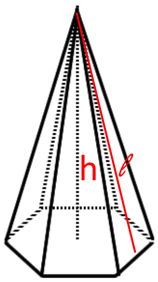
___A___ 5. Face

___C,F___ 6. Vertex



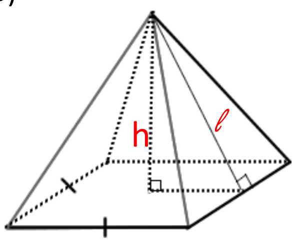
Properly name each pyramid. Label both the height (h) and slant height (ℓ) in each pyramid.

a)



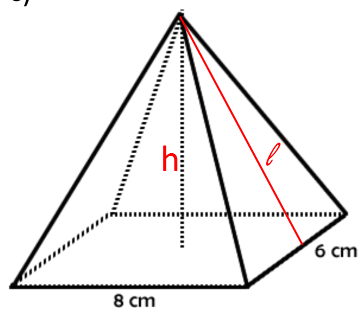
Name: Right hexigonal pyramid

b)



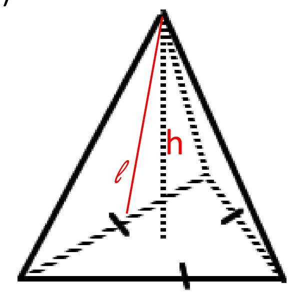
Name: Right square pyramid

c)



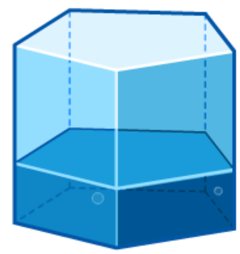
Name: Right rectangular pyramid

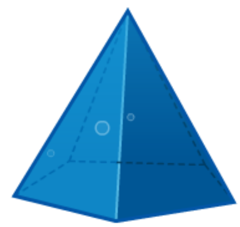
d)



Name: Right triangular pyramid

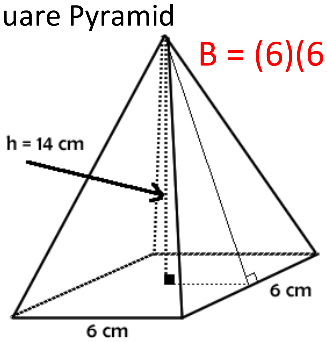
VOLUME$_{\text{PYRAMID}}$ = $\frac{1}{3}$Bh or $\frac{Bh}{3}$





Determine the volume of each pyramid.

a) Square Pyramid



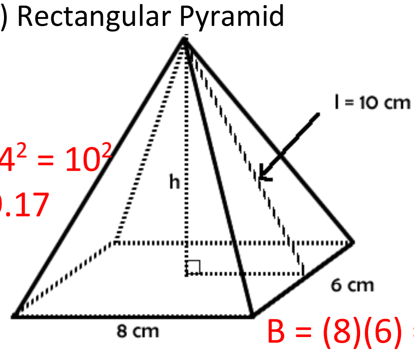
B = (6)(6) = 36

$V = \frac{Bh}{3}$

$V = \frac{(36)(14)}{3}$

Volume = 168 cm^3

b) Rectangular Pyramid



$h^2 + 4^2 = 10^2$

$h \approx 9.17$

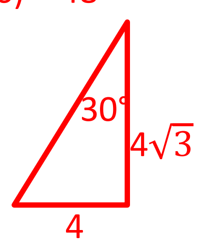
B = (8)(6) = 48

$V = \frac{Bh}{3}$

$V = \frac{(48)(9.17)}{3}$

Volume = 146.72 cm^3 (2 dec.)

c) Regular Hexagonal Pyramid



B = ½ ap

= ½ $(4\sqrt{3})(48)$

= $96\sqrt{3}$

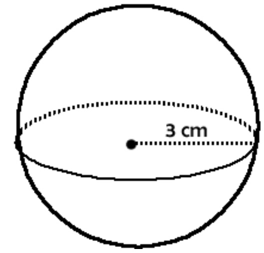
$V = \frac{Bh}{3}$

$V = \frac{(96\sqrt{3})(20)}{3}$

Volume = $640\sqrt{3}$ cm^3

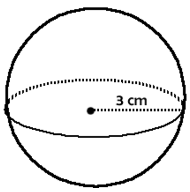
IC10 - SPHERE VOLUME

VOLUME$_{SPHERE}$ = $\frac{4}{3}\pi r^3$ or $\frac{4\pi r^3}{3}$



Determine the volume of each sphere.

a)

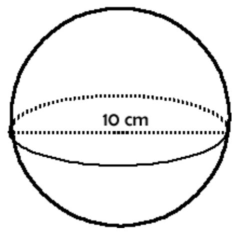


$V = \frac{4}{3}\pi r^3$

$= \frac{4\pi(3)^3}{3}$

$= \frac{108\pi}{3} = 36\pi\ cm^3$

b)



$V = \frac{4}{3}\pi r^3$

$= \frac{4\pi(5)^3}{3}$

$= \frac{500\pi}{3}\ cm^3$

How would you find the volume of a hemisphere?

divide the volume of a sphere in half

$V = \frac{2}{3}\pi r$